

Celebrating 30 EWGLA Meetings: A Personal Retrospective on Location Analysis

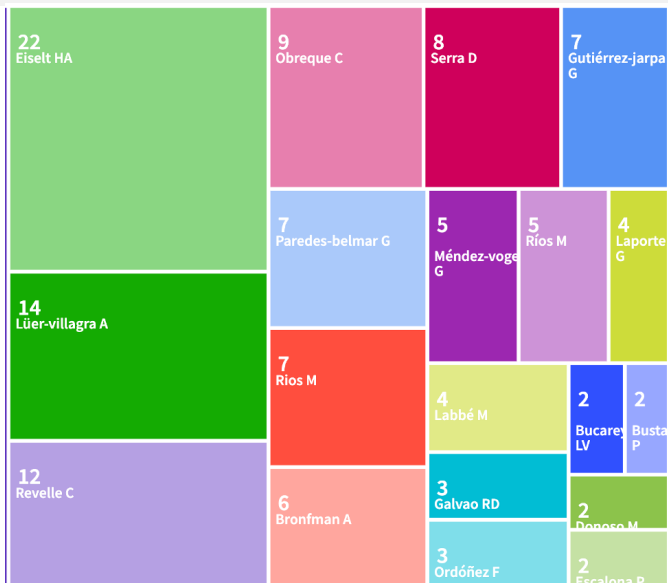
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Most important



Outline

- 1 Some Old Facts
- 2 We're all busy. Take an Aspirin (Ambulances et al.)
- 3 Stand on this line, please (congested immobile servers)
- 4 Come fly with me (competitive hubs)
- 5 Let's get together! (Facility Agglomeration)
- 6 Thoughts (A few)

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Location meetings

- 1st EWGLA, Dijon, France 1985. 12 talks.
From Spain: B. Pelegrin, F. Fernández, J. Moreno.
- The Spanish group in the first 10 years of EWGLA: ≈ 22
- Site: <https://www.euro-online.org/websites/ewgla/ewgla-meetings/>
- (Before that) 1st ISOLDE, Banff, Canada 1978. 42 talks. Every 3 years (except 2011 and 2020):
<https://uwaterloo.ca/isolde-conference/> (Thanks to Sibel Alumur)

EWGLA 3, 1988, Sevilla



Top row: Brian BOFFEY, ?, Jaap VAN DIJK, ?, ?, Justo PUERTO, Dominique PEETERS Middle: Blas PELEGRIN, Françoise ORBAN-FERAUGE, Christian MICHELOT, Jean-Pierre BRANS, ?, Bep & Ken ROSING Front: Jose MORENO, Natividad JIMENEZ, Frank PLASTRIA, Rosa M. RAMOS, Paco R. FERNANDEZ-GARCIA

Status at EWGLA I

- In the 80's:
 - The five "main" location models (p -median, LSCP, MCLP, SPLP, p -center) were well digested
 - Many extensions, theoretical analyses, and applications
- Review by Brandeau & Chiu (1989): "An Overview of Representative Problems in Location Research" 240 references. 50 different problem types. 15 different applications

How did I get into Facility Location

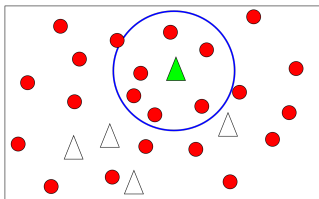
- 1975 - 1985: A telecommunications engineer
- 1985 - 1987: EECS - JHU. Iterative Algorithms.
- 1987 - 1989: DOGEE - JHU, Chuck ReVelle.
- Locating ambulances and firefighting equipment.
- Mostly based on MCLP: Max "covered" demand points. I.e., points with at least 1 vehicle **located** within 8 minutes)
- Hot topic: Sometimes, a vehicle was not **available** within reach

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Location models: Approaches

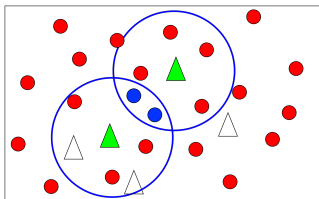
- Deterministic: Hogan & ReVelle (1986). Since the vehicle could be busy, add a second one (backup).
- Marianov & ReVelle (1992), "standard response" for 2 types of fire vehicles (maximize demand with 3 water, 2 ladder).
- Probabilistic way: Formulate a probabilistic model for the system. Requires "busy fraction" q_j of a vehicle (percentage of time busy).



$$q_j = \frac{\bar{\tau} \sum_i n c_i}{24}$$

Location models: Approaches

- Deterministic: Hogan & ReVelle (1986). Since the vehicle could be busy, add a second one (backup).
- Marianov & ReVelle (1992), "standard response" for 2 types of fire vehicles (maximize demand with 3 water, 2 ladder).
- Probabilistic way: Formulate a probabilistic model for the system. Requires "busy fraction" q_j of a vehicle. Impossible before locating!



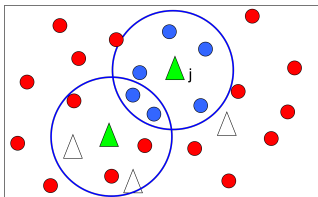
$$q_j = \frac{?}{24}$$

1: Assume busy fractions $q_j = q \quad \forall i$. Independence (Binomial distribution.)

- Maximize $E(\text{coverage})$: MEXCLP - Daskin (1983). The more vehicles cover a demand, the higher its probability of finding one free.
- Maximize demand for which availability is at least α . Find k such that $P(\text{one free of } k \text{ vehicles covering } i) \geq \alpha$. ReVelle & Hogan (1988).
- How do we find true q_j ?

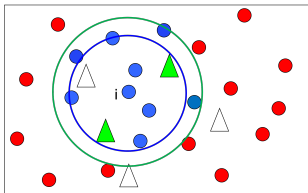
2. Approximated local q_j (or q_i)

- Ball & Lin (1993): q_j , conservative



$$q_j = \frac{\text{Demand within } S}{24}$$

- PLSCP and MALP ReVelle and Hogan (1989). Marianov & ReVelle (1993, 4): firefighting.

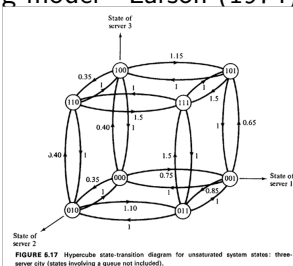


$$q_i = \frac{\bar{\tau} \sum d_i}{24 \sum_{d_{ij} \leq S'} x_j}$$

- What was left? relaxing independence using Queuing!!!

Another world: Queuing descriptive models

- No IP formulations.
- The Hypercube queuing model - Larson (1974) (2^N equations)¹.



- Simplified, Larson (1975). Random vehicle attends calls. N eqns.
- ≈ 10 papers on locating (mostly single) servers: Larson, Berman, Chiu, Batta ... Review: Jamil et al (1999).

¹Includes dispatching. One for each context

Relaxing independence. Joining two worlds

- Batta *et al* (1989) Included Larson(1975)'s "queuing correction factor" in MEXCLP's objective. Corrects the increase in availability when adding a vehicle
- Q-PLSCP, Q-MALP. Marianov & ReVelle (1994, 1996). Uses Erlang B equation². :

$$P(\geq 1 \text{ vehicle free for } i) = 1 - \frac{(1/s!)q_i^s}{\sum_{k=0}^s (1/k!)q_i^k} \geq \alpha$$

²According to Boffey, Galvão, Espejo (2007), the first embedding of queuing in location models

The sad conclusion and further developments

- Berman & Krass (2019) about mobile services, "The underlying queuing models are analytically intractable."
- On top of it: practical aspects
 - Carson & Batta (1990), Repede & Bernardo (1994), Gendreau et al (2001): **daily demand profile, relocation (ambulances)**
 - Serra & Marianov (1998): **multiple scenarios in a day, no relocation (firefighting stations)**
 - Mandell (1998) - **ALS, BLS**, McLay (2009)³ - **MEXCLP + Q + ALS/BLS**
- Practical application: calls with 3 priority levels, reassignment of moving vehicles, 2 vehicle types, multiple demand and traffic scenarios, different crews and shifts. (2024 - 2025). MEXCLP gradual coverage + simulation. See also Jankovič et al, (2024)

³Laura McLay = Laura Albert. Many papers on EMS

Reviews

- INFORMS Tutorial Marianov (2017).
- **Latest review, description, practical aspects: Stratman, Boutilier, Albert (2023). "Uncertainty in Facility Location Models for Emergency Medical Services"**

It was time to change topic

- ISOLDE VI in Lesvos and Xios, 1993. QMALP
- No papers (known to us) on "immobile" congested facilities. And we knew queuing.⁴

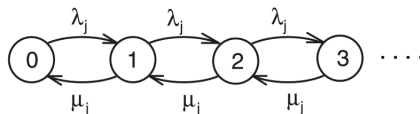
⁴Discovered while looking for references for this talk: Carbone (1974). p -median + chance-constraint, min κ s.t. $P(\text{Gaussian Arrivals at a facility} \leq \kappa) = \alpha$

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Back to chance-constrained and to queuing...

- Marianov & Serra, (1998). p facilities, $M/M/1$. Customers are assigned (or go) to closest. Maximize coverage. We wanted a short queue, or waiting time.



- $P(\text{Queue length at } j \leq b) \geq \alpha \implies \sum_i \lambda_i x_{ij} \leq \mu_j \sqrt[b+2]{(1-\alpha)}$
- $P(\text{Time spent at } j \leq \tau) \geq \alpha \implies \sum_i \lambda_i x_{ij} \leq \mu_j + \frac{1}{\tau} \ln(1-\alpha)^5$

⁵From the cumulative distribution of waiting time

Milking the cow

- Locating p M/M/m/K facilities, (Marianov & Serra, 2002)...

$$\sum_i \lambda_i x_{ij} \leq \mu_j \rho_{\alpha j}$$
- Hierarchical with different policies (Marianov & Serra, 2001),
- M/D/c queues in hubs (from generating function, Marianov & Serra, 2003)
- Health Centers providing two service levels (Marianov, Ríos & Taborga, 2004).

A more real service time distribution: Erlang

- $M/E_r/1/N$ Boffey, Galvão, Marianov (2010) - p -median + constraint
- $M/E_r/m/N$ Marianov, Boffey, Galvão (2009) - p -median + constraint

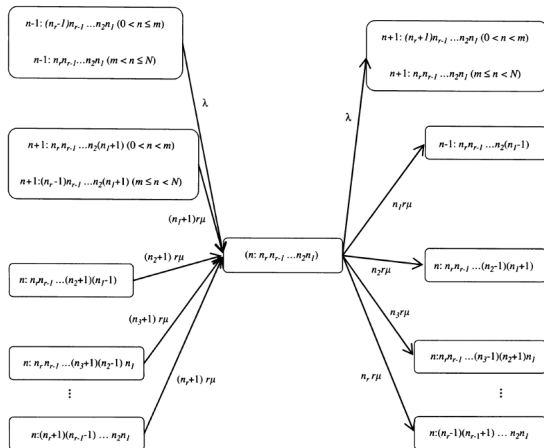


Figure 1 State transition diagram.

Demand elastic to time and waiting: customers' choice

- All our previous papers assumed that customers would go to their closest facility. We want them to choose by (travel + waiting).
- $\uparrow$ users $\implies \uparrow$ waiting $\implies \downarrow$ users: Equilibrium equations!
- (non-competitive) Maximize expected demand (λ) (Marianov, 2003; Marianov et al, 2005). β_{ijn} is elasticity to travel and n in queue.

$$\lambda_{ij} = \bar{\lambda}_i \left(\sum_n \beta_{ijn} P_n(\lambda_j) \right) x_{ij}$$

- (competitive) Logit choice rule (Marianov et al, 2008). Logit utility:

$$u_{ij} = -\gamma t_{ij} - (1 - \gamma) w_j$$

Immobile congested reviews, contemporary works

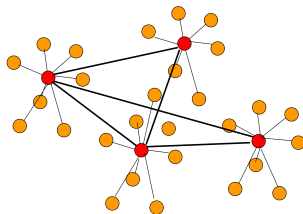
- Wang et al (2002), Berman & Drezner (2007) Min (travel + waiting), capacities
- Aboolian et al (2007) Min approximated (travel + waiting)
- Aboolian et al (2009), Castillo et al (2009) Min Max (travel + waiting), social optimum
- Drezner & Drezner (2011) Elastic demand, equilibrium
- Aboolian et al (2022a & b), Krass et al (2023) Aboolian & Karimi (2025): ϵ -optimal.
- Good reviews by Berman & Krass (2019), Aboolian and Karimi (2023)⁶
- ...and in parallel with this...

⁶cite some 25 papers on immobile congested servers.

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The hub location problem

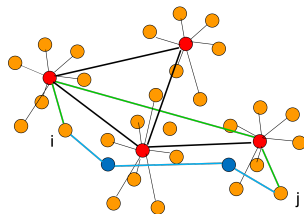


- Goldman (1969) proved "Node property of processing centers". First formulations of p -hub location: O'Kelly (1986, 1987)
- O'Kelly (1992) and Campbell (1994) link to location problems
- 106 papers cited in Campbell et al (2002). Only one paper on competitive hub location, in 1999.⁷

⁷Skorin-Kapov (1998) first with customer choice! Not location.

Competitive hub location problem

- It was us! Marianov, Serra, ReVelle, (1999)



- MAXCAP, node demands $\rightarrow$ origin-destination flow demands. Binary rule⁸.

⁸Sasaki published a paper in 1999 on competitive hub networks in Japanese, the Nanzan Management Review. I learned this last week!

Competitive hub location, Customers can choose different paths at different times

- Sasaki & Fukushima (2001): first von Stackelberg: continuous, Logit rule, gradient method. One hub each (1 Leader, p disjoint followers).
- Sasaki (2005): (r/p) , discrete, enumeration and greedy.⁹
- Eiselt, Marianov, (2008): Gravity rule, first combination of cost and time. Maximize market share.

$$u_p = \frac{A_p}{\gamma t_p^\beta + (1 - \gamma)c_p^\beta}$$

- Competitive hub location & pricing (Lüer-Villagra & Marianov, 2013)

⁹Both in Japanese journals

Competitive Hub location, application

- Application to P & R in New York (Aros-Vera, Marianov, Mitchell, 2013)
 - Competes against private car
 - First Gravity then Logit customer choice rule
 - Linearization of Logit (was in a Working paper by Haase, 2009)

Competitive hub locations, selected contributions

- Adler (2001), Adler & Smilowitz (2007) price-location competition
- Sasaki, Campbell, Ernst, Krishnamoorthy (2014): competitive leader-follower hub-arc.
- Mahmutogullari and Kara (2016) First MIP for Leader-follower
- Andrade de Araújo et al (2020) B&C for the Leader-follower.
- Espejo, Marín, Muñoz-Ocaña, Rodríguez-Chía (2023) New B&C
- Domínguez-Bravo, Fernández, Lüer-Villagra (2024) competitive and congested.
- Parvasi et al (2025) follower locates and set prices, incumbent changes prices. Latest references.
- Good reviews: Contreras and O'Kelly (2019) in Laporte, et al book, O'Kelly et al (2025) (also Alumur, Campbell, Contreras, Kara, O'Kelly (2021))

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Competitive problems: fast forward from 1929 to 2025

- Hotelling, (1929): two vendors on a line ("ice cream vendors on a beach")
- Hakimi (1983) Centroid and Medianoid (Leader and Follower, resp.)
- ReVelle (1986) Hotelling on a Network
- A HUGE amount of literature
 - Large Spanish group: Arrondo, Campos-Rodríguez, Cánovas, Carrizosa, Delgado-Gómez, Dorta-González, Elizalde, (Fernández - Elena, Pascual, Francisco), García-Pérez, García-Fernández, Moreno-Pérez, Ortigosa, Pelegrin, Redondo, Ruiz-Hernández, Santos-Peñate, Serra, Suárez-Vega
 - Tammy Drezner (and Zvi), Eiselt, Aboolian, Berman, Krass, Hodgson, Benati, Hansen, Labbé, Laporte, de Palma, Thisse, Narula, Plastria, Younies.
 - Last reviews: **Drezner & Eiselt (2024)**; Eiselt, Marianov, Drezner(T) (2019), Ashtiani (2016)

There are still challenges in competitive problems

- Spatial interaction: Customers are more attracted to a facility if it is in a cluster (Many economists, geographers, transportation, and marketing researchers)
- Clusters mean more opportunities (Multi-purpose shopping and comparison shopping)
- $\implies$ facilities clusterize (**Agglomeration**)^{10 11 12}

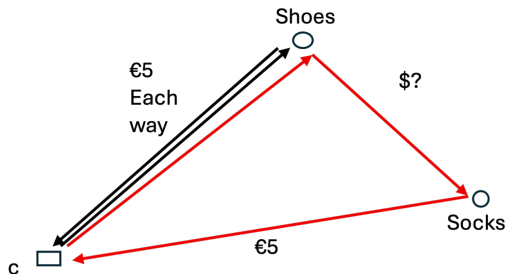
¹⁰McLafferty & Gosh (1986) conclude dispersion on a line!

¹¹O'Kelly (1981), (1983) compares single and multiple-shopping choice models on data.

¹²Marianov & Eiselt (2016) ANOR edited by Dolores Santos-Peñate, 10th Anniversary of the Spanish Location Network.

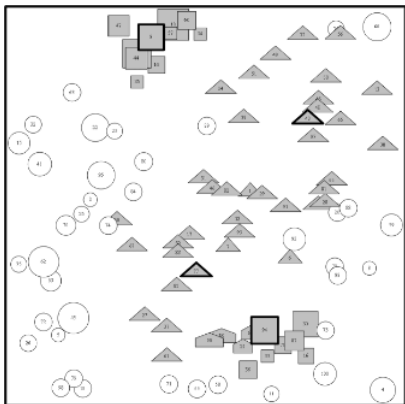
Multi-purpose and comparison shopping¹³

- Marianov, Eiselt, Lüer-Villagra (2018) MP, duopoly, binary rule.
- Suppose all customers have a willingness to spend €20 in travel to purchase shoes and socks

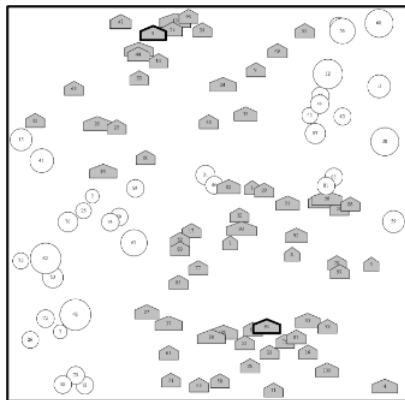


- Locating close to the incumbent increases the market of both!!!
(Entrant more than Incumbent)

¹³John Hodgson (1981) did the first formulation of a location model for trips with more than one stop



Square: 1,311, Triangle: 2,297

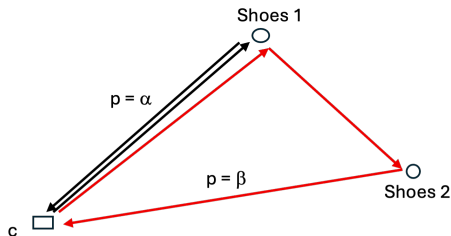


Square = Triangle: 3,270

- Lürer-Villagra, Marianov, Eiselt (2022) - Leader-follower;
Méndez-Vogel et al., (2023) - MP Partially binary Logit
- good papers on MP: T. & Z. Drezner, O'Kelly, Kalczynski (2023 -)
- > 2 products (Miklas-Kalczynska, 2024), Khapugin, Melnikov (2019)

Comparison shopping

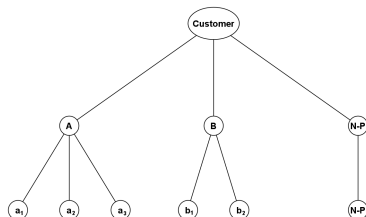
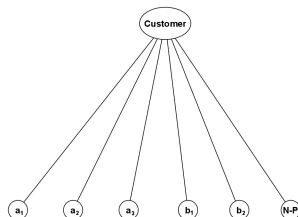
- Marianov, Eiselt, Lüer-Villagra (2020).



- $P(\text{purchase single trip}) = \alpha$, $P(\text{purchase comparison trip}) = \beta > \alpha$
- $\implies$ facilities cluster to make it easier to compare and increase sales.
Or, they locate far enough to have their monopolistic markets.

Agglomeration means sequential decisions and correlation

- Popular choice rules (e.g. Logit) represent one-stage decisions
- Many decisions are sequential. Nested Logit. Allows correlation. Was assumed non-convex!



- Méndez-Vogel et al (2023b): Single trip, submodularity + B&C;
- Méndez-Vogel, Marianov, Fernández, Pelegrin, Lüer-Villagra, (2024) Multilevel Nested, comparison shopping: (geographical nests).
- Last sequel: He et al (2025) Leader-follower, one nest per firm

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Some thoughts

- Since the 1990's, people have prophesied the death of location modeling, BUT ... (Problems become more complex). A more complete view: Marianov & Eiselt (2024): "50 'Tears' of Location Theory ..."
- Emergency vehicles: Converged to context-tailored methods
- Congestion + consumers' choice: Customers are assumed to know the current state of all facilities
- Competitive hubs: Inclusion of frequencies and timely route connections
- Competitive problems: Better models of customers' behavior. Use of big data, predictive tools, and spatial interaction